# SCALED SIMULATION DESIGN OF HIGH QUALITY LASER WAKEFIELD ACCELERATOR STAGES \*

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#### Abstract

Design of efficient, high gradient laser driven wakefield accelerator (LWFA) stages using explicit particle-incell simulations with physical parameters scaled by plasma density is presented. LWFAs produce few percent energy spread electron bunches at 0.1-1 GeV with high accelerating gradients. Design tools are now required to predict and improve performance and efficiency of future LWFA stages. Scaling physical parameters extends the reach of explicit simulations to address applications including 10 GeV stages and stages for radiation sources, and accurately resolves deep laser depletion to evaluate efficient stages.

# **INTRODUCTION**

LWFAs achieve accelerating electric fields thousands of times those of conventional accelerators by using the radiation pressure of an intense laser to drive a plasma wave (review: [1]), and quasi-monoenergetic beams have recently been demonstrated [2, 3]. Experiments are now being designed to control injection and accelerator structure to increase efficiency and beam quality. Applications include 10 GeV modules for a high energy physics collider [4], and efficient high quality accelerators near 0.5 GeV for Thomson gamma sources in nuclear security [5].

As it drives the wake, the laser pulse is simultaneously shaped by its interaction with the plasma, and a stable accelerating structure requires balancing this process, making self-consistent simulations important but challenging. Resolving the laser wavelength ( $\sim \mu m$ ) over the acceleration distance and wake volume drives computational load. Time-explicit three dimensional particle-in-cell simulations which resolve the laser period, the most direct approach, require Mhours to simulate cm-scale GeV experiments [6]. Bunch energy scales approximately as the square of the plasma wavelength  $\lambda_p = \sqrt{\pi c^2 m/e^2 n_e}$ , limited by dephasing of particles from the wake and laser depletion, which predicts 10 GeV stages will operate in mscale plasmas at order  $10^{17}$ /cc densities. Because wake volume (resolving a wake period) and acceleration length (dephasing and depletion limits) each scale roughly as  $\lambda_n^3$ , simulation cost scales as the 6th power of  $\lambda_p$ , or the third power of the beam energy. Simulating 10 GeV stages explicitly would then take on the order of a billion processor

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hours, which is beyond state of the art at this writing.

Here we show that simulations at high densities can be used by scaling physical parameters to predict performance of low density, high energy LWFA stages in the quasilinear regime, e.g. the highest wake amplitude (gradient) for which the wake remains approximately sinusoidal, typically near  $a_0 \sim 1$ , with  $a_0$  the dimensionless laser amplitude [1]. This allows nearly symmetric acceleration of externally injected electrons and positrons, and the fields can be shaped using the laser profile to control beam emittance [7]. Previous work showed that scaled simulations can accurately represent beam loading and fields, and used them to predict design of 10 GeV stages [8]. Here, we show that such simulations accurately resolve quasilinear field structure, laser depletion, many aspects of laser evolution, and energy gain and spectrum. Simulations in conjunction with theory are used to design efficient stages by selecting laser pulse length, spot size, and other parameters.

# RESULTS

Scaled simulations are motivated by noting that while analytic solutions of the wake excitation and laser propagation and depletion are not available in the nonlinear, self consistent, multidimensional regime, the plasma response scales with density  $n_0$  in both the linear and nonlinear regimes. In particular, we note that in linear theory the wake structure remains constant if the laser pulse length and width are constant in  $\lambda_p$  units, and that the nonlinear self focusing parameter  $\sim a_0^2 (k_p w_0)^2$  is constant if the laser spotsize  $w_0$  is scaled with  $\lambda_p$ . Simulations were then performed, using the VORPAL framework developed by Tech-X [9], at various 'scaled' densities but with the laser parameters  $a_0$ ,  $k_p w_0$ ,  $k_p L$  constant [8], with L the length of the Gaussian laser pulse. The dimensions of externally injected particle bunches were also scaled to the plasma wavelength and their density with  $n_0$ .

Linear fluid theory (Fig. 1), shows the tradeoffs governing choice of spot size. At small  $k_p w_0$ , transverse fields absorb an increasing fraction of laser energy, reducing efficiency. The laser pulse is guided by a plasma channel scaled to guide the spot. Also at small  $k_p w_0$  the channel dispersion reduces laser group velocity, reducing the dephasing length and hence stage energy. On the other hand, at  $a_0 \sim 1$  large spot sizes cause power to exceed the critical power for relativistic self focusing, causing the pulse to focus and enter the blow-out regime. These criteria define an optimal range of roughly  $4 < k_p w_0 < 6$  (green). Additionally, this range produces a relatively uniform wake

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Figure 1: Linear dephasing length  $L_{deph}$ , energy ratio in the focusing to accelerating field, and for  $a_0 = 1$  power normalized to critical power for self focusing  $(P/P_c)$ (lines). Simulation field ratios (\*) and normalized dephasing lengths (boxes) for the  $k_p w_0 = 1$  case in Fig. 3.

transversely which can be beneficial for beam dynamics and emittance.

Simulations (Fig. 2) show that wake structure and amplitude scale as predicted by theory over two orders of magnitude in  $n_0$ , and that 2D slab geometry and 3D axisymetric wakes (resolving radius and propagation, with symmetry in  $\theta$ ) have the same form at the percent level. The spot size is  $k_p w_0 = 5.3$ , near the center of the range shown in Fig. 1,  $a_0 = 1$ , and  $k_p L = 2$  (optimal from linear scalings with  $a_0$ constant). Note that the quasistatic algorithm in WAKE is a reduced model which does not resolve the fast laser oscillation, and also assumes slow laser evolution. Remaining differences appear to be primarily due to differences in laser pulse shape specification. Similar comparisons were obtained using explicit and envelope [10] models in VOR-PAL. The simulations show linear scalings for the wake hold in the quasi-linear regime, and benchmark the various algorithms used. They determine wake amplitude and multidimensional wake structure not determined by theory.

Simulation over a dephasing length at various  $n_0$  showed that laser evolution, self focusing, guiding, depletion, and beam dephasing scale as anticipated. Energy gain scaled with  $\sim \lambda_p^2$  as expected, with 100 MeV observed at  $n_0 = 10^{19}$  and 1100 MeV at  $n_0 = 10^{18}$ , with laser energy scal-



Figure 2: Wake density contours of scaled VORPAL explicit simulations  $n_0 = 10^{19}/\text{cc}$  in 2D slab geometry (black) and WAKE quasistatic simulations at  $n_0 = 10^{17}/\text{cc}$  in axisymmetric geometry (red dashed).

**Advanced Concepts** 

ing as  $\lambda_p^3$  [8]. This predicts 10 GeV gain for test particles at  $n_0 = 10^{17}$ /cc using a 40 J, 120 fs laser. Energy is somewhat greater than that predicted by analytic theory because of wake nonlinearity. Power is 90% of the self-focusing power, and detuning channel density gradient 40-50% relative to low power matching was required to achieve propagation with less than 10% spot size oscillation.

Laser and wake evolution during propagation were evaluated for a range of pulse lengths  $0.5 < k_pL < 3$  (Fig. 3), for fixed pulse energy. This tuning mode is typical of laser operations, and also stays on the threshold quasilinear operation since  $a_0$  then rises for short pulses where self focusing is less effective. The structure is optimized near  $k_pL = 1$  ( $a_0 = 1.4$ ) where field is increased  $\sim 40\%$  relative to  $k_pL = 2$  and where the pulse depletes its energy and accelerating field rolls off at the dephasing length. Shorter (longer) pulses result in depletion before (after) dephasing, reducing efficiency. Scaling of field amplitude, evolution, and depletion with  $n_0$  was reasonably consistent with theory. Depletion is slightly better at low  $n_0$ , consistent with the energy scalings above.

Simulations at  $k_pL = 1$  and varying  $k_pw_0$  verified the predicted trade-offs with  $k_pw_0$  (Fig. 1). Transverse wake energy is significantly greater than linear theory at  $k_pw_0 = 3$  because strongly nonlinear wake curvature and blow out occur. Curvature also extends dephasing  $\sim 10\%$ , and the  $L_{deph}$  plot is normalized to the linear result at  $k_pw_0 = 7$  to show scaling. Multiple points at  $k_pw_0 = 5.3$ show variation over 3x in numerical resolution and 4x in density. Self focusing modulated the spotsize significantly for  $k_pw_0 \ge 7$ .

There is a small 'ripple' in the accelerating field due to laser spotsize oscillation from imperfect matching into the channel. Its period changes with  $n_0$  because the ratio of laser focal depth (and electron beam betatron period) to dephasing length is not constant. Because the laser is nearly matched, this does not significantly affect predictions in this case. The scaled simulations have the asset of resolving deep depletion of the laser pulse which can be



Figure 3: Laser energy (lines) and accelerating field (triangles) evolution versus propagation for various pulse lengths  $k_pL$ . At  $k_pL = 1$  and 2, density scaling is shown. At  $n_e 10^{19}$ /cc,  $L_{dephasing} = 970 \ \mu$ m and  $E_0 = 300 \text{ GV/m}$ .

A14 - Advanced Concepts



Figure 4: Laser energy evolution and resulting beam loaded bunch energy gain for a tapered stage at  $k_p L = 1$  is shown for various numerical resolutions and in 2D and 3D.

problematic for other reduced algorithms. Multiple models are needed to comprehensively address designs for 10 GeV and beyond. Lorentz boosted (Vay et al. these proceedings) and envelope [10] simulations have also been conducted for these parameters.

Acceleration of high charge bunches, with  $\approx 50\%$  loading of the wake, was simulated next in the  $k_p L = 1$  case. We choose the bunch length to make the accelerating field within the bunch constant at a given wake phase, and use a linear increase in axial plasma density (tapering) to reduce phase slippage as detailed in [8]. By shortening  $\lambda_p$  over the propagation length, tapering maintains phase as the relativistic electrons slip forward relative to the laser, increasing gain and reducing energy spread. Fig. 4 shows the laser energy depletion and resulting particle bunch energy gain at several numerical resolutions, indicating convergence at the percent level up to the dephasing point. With tapering, electric field increases with density, partially compensating laser depletion and allowing extraction of a greater fraction of the laser energy, close to 50%. Depletion is limited by pulse lengthening as the laser red shifts and broadens (similar to 1D observations in [11]). Because the stage operates in the  $k_p w_0 = 5.3$  regime (Fig. 1), almost all of the depleted energy is usable as accelerating field. In this regime, 3D depletion is mildly stronger than 2D as reflected in Fig. 1, which roughly counters the density scaling inaccuracy noted above in prediction of stage performance at 10 GeV. Self focusing is also mildly stronger in 3D which can require channel profile adjustment by  $\sim 10\%$ .

Electron bunch energy spread was reasonably converged at  $2.5 < \Delta E/E < 4\%$  for the parameters of Fig. 4, and scaled simulations have hence been used to study beam quality as well as efficiency [8]. By tuning emittance matching into the structure, 1% level energy spread was achieved (Fig. 5). Scaled energy gain is 9 GeV and charge is ~ 300 pC for a 40 J, PW-class laser operating at 60 fs FWHM pulselength, such as the proposed BELLA laser at the LOASIS laboratory of LBNL. Such stages are of interest as modules in an eventual LWFA collider [4]. For parameters relevant for nuclear material gamma sources



Figure 5: Energy spectrum of a stage with laser-plasma parameters as in Fig. 4, showing 1.5% integrated and subpercent slice energy spread by tuning bunch injection.

 $k_p L = 2$  provides better matching to available lasers, and 0.4 GeV gain and ~ 50 pC charge is predicted using a 0.5 J, 25 fs laser. Work is in progress on emittance [7] and beam load matching the bunch to the wake structure to further increase efficiency and quality.

### CONCLUSION

We have presented density scaled explicit particle in cell simulations, showing that these can quantitatively predict wake structure and fields, laser depletion and evolution, and particle energy gain and spread of high energy LWFA stages using computationally tractable simulations at higher densities. Their ability to resolve deep laser depletion makes scaled simulations a useful complement to other reduced simulation techniques to provide comprehensive simulation of high energy stages. A regime of efficient acceleration and good energy spread was presented. Moreover, the simulations demonstrate that linear scalings extend into the quasilinear regime of  $a_0 \sim 1$ , and the demonstrated scaling allows a single simulation (which captures self consistent structure and evolution not included in linear theory) to predict behavior over a wide range of densities/laser powers.

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